📘 Assignment: Probability & Bayes’ Theorem

Objective: Strengthen the understanding of classical probability and conditional reasoning through Bayes’ Theorem.

Total Questions: 10

Chapters Covered:

Classical & Conditional Probability

Independent Events

Total Probability

Bayes’ Theorem with real-life use cases

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✅ Instructions:

Attempt all questions showing all steps and formulas used.

Round off answers to 2 decimal places where needed.

Diagrams/Venn may be used wherever necessary.

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🔢 PART A: PROBABILITY (Q1–Q6)

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Q1. Coin and Dice

A fair coin is tossed and a fair 6-sided die is rolled.

👉 Find the probability:

a) Getting a head and an even number

-> The probability of getting a head (H) on a coin toss is:

P(H) = 1 / 2

The probability of getting an even number (E) on a die roll is:

P(E) = 3 / 6 = 1 / 2 (since there are 3 even numbers: 2, 4, and 6)

Assuming the coin toss and die roll are independent events, the probability of getting both a head and an even number is:

P (H ∩ E) = P(H) × P(E)

= (1 / 2) × (1 / 2)

= 1 / 4

So, the probability of getting a head and an even number is 1 / 4 or 25%.

b) Getting a tail or a number greater than 4

-> The probability of getting a tail (T) on a coin toss is:

P(T) = 1 / 2

The probability of getting a number greater than 4 (GT4) is:

P(GT4) = 2 / 6 = 1 / 3

Probability of neither event happening = (1 / 2) × (2 / 3) = 1 / 3

Probability of at least one event happening = 1 - Probability of neither event happening

= 1 – 1 / 3 = 2 / 3

c) Not getting head and number less than 4

-> The probability of not getting head (getting tail) is:

P(NH) = 1 / 2

The probability of getting a number less than 4 is:

P(LT4) = 3 / 6 = 1 / 2

Assuming the coin toss and die roll are independent events, the probability of getting both a not head (tails) and a number less than 4 is:

P (H ∩ E) = P(H) × P(E)

= (1 / 2) x (1 / 2)

= (1 / 4)

Q2. Deck of Cards

From a well-shuffled standard deck of 52 cards, one card is drawn at random.

👉 Find the probability of:

a) Drawing a black face card

-> There are 6 black face cards in a standard deck of 52 cards (2 suits: spades and clubs, each with 3 face cards: King, Queen, Jack). Probability of getting a black face card is:

P(B) = 6 / 52 = 3 / 26

b) Drawing a card that is a diamond or a king

-> There are 13 diamond cards and 3 king cards (excluding the diamond) in the standard deck of 52 cards.

So, the probability is:

= 16 / 52 = 8 / 26 = 4 / 13

c) Drawing a non-ace card

There are 4 ace cards in a standard deck of 52 cards.

Number of non-ace cards = 52 - 4 = 48

Probability = Number of favorable outcomes / Total number of outcomes

= 48 / 52 = 24 / 26 = 12 / 13

Q3. Conditional Probability

In a group of 100 people:

60 like tea

30 like both tea and coffee

20 like only coffee

👉 Find the probability that a person:

a) Likes coffee given they like tea

-> This is a conditional probability problem. We want to find P(Coffee|Tea).

P(Coffee|Tea) = Number of people who like both tea and coffee / Number of people who like tea

= 30 / 60

= ½

b) Likes tea or coffee

-> Number of people who like tea or coffee = Number of people who like only tea + Number of people who like only coffee + Number of people who like both

= 30 + 20 + 30

= 80

Probability = Number of people who like tea or coffee / Total number of people

= 80 / 100

= 4/5

c) Likes neither

-> Number of people who like neither = Total number of people - Number of people who like tea or coffee

= 100 - 80

= 20

Probability = Number of people who like neither / Total number of people

= 20 / 100

= 1/5

Q4. Probability from Table

A bag contains 5 red balls, 4 green balls, and 3 blue balls. One ball is drawn at random.

👉 Find:

a) The probability it is not green

-> Total number of balls = 5 + 4 + 3 = 12

Total Red and Blue Balls = 12 – 4 = 8

Probability = 8 / 12 = 2 / 3

b) The probability it is either red or blue

-> Number of red or blue balls = 5 (red) + 3 (blue) = 8

Probability = Number of red or blue balls / Total number of balls

= 8 / 12

= 2 / 3

c) If two balls are drawn without replacement, what's the probability both are red?

-> Probability of first ball being red = 5 / 12

If the first ball is red, there are now 4 red balls and 11 total balls.

Probability of second ball being red = 4 / 11

Probability of both balls being red = (5 / 12) × (4 / 11)

= 20 / 132

= 5 / 33

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Q5. Checking for Independence

Events A and B have the following probabilities:

P(A) = 0.6, P(B) = 0.5, P(A ∩ B) = 0.3

👉 Are events A and B independent? Show the formula and reasoning.

-> Formula for Independence:

P(A ∩ B) = P(A) × P(B)

Given Probabilities:

P(A) = 0.6

P(B) = 0.5

P(A ∩ B) = 0.3

Checking for Independence:

P(A) × P(B) = 0.6 × 0.5

= 0.3

Since P(A ∩ B) = P(A) × P(B) = 0.3, events A and B are independent.

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Q6. Total Probability Theorem

A school has 40% students from City A, 35% from City B, and 25% from City C. The probabilities of attending class on a rainy day are:

City A: 0.7

City B: 0.6

City C: 0.9

👉 What is the probability that a randomly selected student attends class on a rainy day?

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Given Probabilities:

P(A) = 0.4 (probability of a student being from City A)

P(B) = 0.35 (probability of a student being from City B)

P(C) = 0.25 (probability of a student being from City C)

P(Attends|A) = 0.7 (probability of attending class given from City A)

P(Attends|B) = 0.6 (probability of attending class given from City B)

P(Attends|C) = 0.9 (probability of attending class given from City C)

Total Probability Theorem:

P(Attends) = P(A) × P(Attends|A) + P(B) × P(Attends|B) + P(C) × P(Attends|C)

= 0.4 × 0.7 + 0.35 × 0.6 + 0.25 × 0.9

= 0.28 + 0.21 + 0.225

= 0.715

The probability that a randomly selected student attends class on a rainy day is 0.715 or 71.5%.

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🔢 PART B: BAYES’ THEOREM (Q7–Q10)

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Q7. Basic Bayes' Application

A factory has two machines:

Machine A produces 60% of the products and has 2% defect rate.

Machine B produces 40% of the products and has 3% defect rate.

👉 If a product is defective, what is the probability it came from Machine B?

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Given :

Machine B Products = 40%

Defect rate of Machine B Products = 3%

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Q8. Medical Diagnosis Problem

In a population, 1% of people have a disease.

A test detects it correctly 99% of the time (true positive rate). It also gives a false positive 5% of the time.

👉 If a person tests positive, calculate the probability that they actually have the disease. Use Bayes’ Theorem.

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Q9. Email Classification

A spam filter is tested on a batch of emails:

30% of emails are spam

The word “offer” appears in 80% of spam and 10% of non-spam emails

👉 If an email contains “offer”, what is the probability it is actually spam?

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Q10. Student Major Identification

At a university:

40% of students are Engineering majors

30% are Business majors

30% are Arts majors

The probability that a student knows Python is:

0.8 if Engineering

0.3 if Business

0.2 if Arts

👉 If a student is selected at random and they know Python, what is the probability they are from Engineering?